(c) $\frac{t_n}{n(n+1)} = \dots = \frac{n}{n+1}\sigma_n - \frac{n-1}{n}\sigma_{n-1}$, so

$$\sum_{k=1}^{n} \frac{t_k}{k(k+1)} = \frac{n}{n+1}\sigma_n .$$

So if $\exists \lim \sigma_n$, then $\exists \sum_{1}^{\infty} \frac{t_k}{k(k+1)}$, and vice versa.

8.42 Converges for $\forall x \in \mathbb{R}$. Indeed,

(a) $x \neq m\pi$, $\prod_{k=1}^{n} \cos\left(\frac{x}{2^k}\right) = \frac{2^n \sin\frac{x}{2^n}}{2^n \sin\frac{x}{2^n}} \prod_{k=1}^{n} \cos\left(\frac{x}{2^k}\right)$

$$= \frac{\sin x}{2^n \sin\frac{x}{2^n}} \longrightarrow \frac{\sin x}{x}$$

(b) $x = m\pi$, $m \in \mathbb{Z}$. If $m = 0$, converges to 1.
Otherwise, pick N s.t. for $n \geq N$, $\sin\frac{x}{2^n} \neq 0$.

Then
$$\prod_{k=1}^{n} \cos\left(\frac{x}{2^k}\right) = \underbrace{\prod_{k=1}^{N-1} \cos\left(\frac{x}{2^k}\right)}_{\text{this does not depend on } n} \underbrace{\prod_{N}^{n} \cos\left(\frac{x}{2^k}\right)}_{\frac{\sin(x/2^{N-1})}{2^{n-N+1}\sin\left(\frac{x}{2^n}\right)}}$$

$$\downarrow$$

$$\frac{\sin(x/2^{N-1})}{x/2^{N-1}}$$

So LHS converges as $n \to \infty$. $\blacksquare$