HW 2 (4001)

8.15 (g)

$$\frac{1}{n\log\left(1+\frac{1}{n}\right)} = \frac{1}{\log\left(1+\frac{1}{n}\right)^n} \longrightarrow 1$$

So it does not conv.

8.23 Suppose $\sum n a_n$ converges. Then, by Dirichlet,

$$\sum \frac{1}{n} n a_n = \sum a_n \text{ converges; contradiction.}$$

8.24 Use $\sqrt{pq} \le \frac{p+q}{2}$.

$$\sum \sqrt{a_n a_{n+1}} \le \sum \frac{a_n + a_{n+1}}{2} = \frac{1}{2}\left(\sum a_n + \sum a_{n+1}\right)$$

If, e.g. $a_n \nearrow$, then $a_n \le \sqrt{a_n a_{n+1}} \le a_{n+1}$ and use comparison.

8.26 Enough for $x \ne 2m\pi$. (o/w converges)
Let

$$a_k := \frac{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}}{k}, \quad b_k = \sin kx.$$

Show that converges for all $x \ne 2m\pi$ by applying Dirichlet. Show $a_k \searrow 0$, and use that

$$\left|\sum_1^n b_k\right| \le \left|\frac{1}{\sin\left(\frac{x}{2}\right)}\right|.$$

($a_k \to 0$ b/c it is a Cesaro limit of the $\frac{1}{k}$'s)