Final, MATH 4001 — Solutions to Y/N Questions
$10 \cdot 2 = 20$ points
December 11, 2015

YOUR NAME: ....

Circle either Y or N (but not both) at the end of each question. No explanation is needed.

1. A conditionally convergent series can always be rearranged so that its sum becomes 1. *Y*
2. The union of countably many zero-sets is also a zero set. *Y*
3. The product
$$\prod_{n\geq 1}\left(1+\frac{1}{n^{4/3}}\right)$$
is convergent. *Y*
4. If I is an arbitrary interval, $f_n \in L(I)$, $|f_n| < K$ a.e. for all $n \geq 1$, and $f_n \to f$ as $n \to \infty$ a.e. on I, then $\int_I f_n(x)\,dx \to \int_I f(x)\,dx$ as $n \to \infty$, where the integrals are Lebesgue integrals. *N*
5. Let $\sum_{n=0}^{\infty} a_n, \sum_{n=0}^{\infty} b_n$ be two conditionally convergent series, such that the sum of each one is 3. Let $\sum_{n=0}^{\infty} c_n$ denote their Cauchy product. If $\sum_{n=0}^{\infty} c_n$ converges at all, then its sum must be 9. *Y*
6. Let $f_n \in L(I)$ for all $n \geq 1$, $f_n \to f$ as $n \to \infty$ a.e. on I. Suppose that we cannot find a dominating function in $L(I)$ for the family $\{f_n\}_{n\geq 1}$, but there is a $g \in L(I)$, such that $|f| \leq g$ a.e. on I. Then it is still true that $f \in L(I)$. *Y*
7. Let $f(x) := \sum_{n=0}^{\infty} a_n x^n$ for $-1 < x < 1$ (we assume convergence in that interval) and suppose that $\sum_{n=0}^{\infty} a_n = 0$. Then it follows that $\lim_{x\to 1-} f(x) = 0$. *Y*
8. If f is (improper) Riemann-integrable on $[1, \infty)$, then it is also Lebesgue integrable there. *N*
9. If $f(x) = \sum_0^{\infty} a_n x^n$ for $-1 < x < 1$ (we assume convergence in that interval) and $\lim_{x\to 1-} = S$, then it follows that $\sum_0^{\infty} a_n = S$. *N*
10. If f is Riemann integrable on $[0, 1]$, then it is an upper function, i.e. $f \in U([0, 1])$. *Y*