Math 4200 Introduction to Topology
Homework Set 6
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Problem 1: Prove that if $X$ is a totally ordered set in which every closed interval is compact, then $X$ has the least upper bound property. (4P)

Problem 2: Let $(X, d)$ be a metric space and $A \subset X$ be nonempty.

   a) Show that $d(x, A) = 0$ if and only if $x \in A$.

   b) Show that if $A$ is compact, $d(x, A) = d(x, a)$ for some $a \in A$. (6P)

Problem 3: Consider the topology $\mathcal{O}_2$ on $\mathbb{R}$ from homework 3.

   a) Show that $[0, 1]$ is not compact with respect to the topology $\mathcal{O}_2$.

   b) Show that $(\mathbb{R}, \mathcal{O}_2)$ is connected, but not path-connected. (6P)