THE WARP DRIVE AND ANTIGRAVITY

Homer G. Ellis

Abstract. *The warp drive envisioned by Alcubierre that can move a spaceship faster than light can, with modification, levitate it as if it were lighter than light, even allow it to go below a black hole's horizon and return unscathed. Wormhole-like versions of the author's 'drainhole' (1973)* might *provide the drive, in the form of a by-pass of the spaceship composed of a multitude of tiny topological tunnels. The by-pass would divert the gravitational 'ether' into a sink covering part of the spaceship's hull, connected by the tunnels to a source covering the remainder of the hull, to produce an ether flow like that of a river that disappears underground only to spring forth at a point downstream. This diversion would effectively shield the spaceship from external gravity.*

In a letter that appeared in 1994 in the journal *Classical and Quantum Gravity* Alcubierre exhibited a space-time metric that describes a surprising phenomenon occurring in a flat, euclidean space: a spherical region of the space glides along geodesically with a prescribed velocity $\mathbf{v}_{\mathrm{s}}(t)$ as if it were a (practically) rigid body unattached to the remainder of space [1]. The velocity, directed along the x axis, is arbitrary as to magnitude and time dependence. In particular, the speed of the moving region can be anything from zero to many times the speed of a light pulse traveling on a parallel track outside the sphere. The ability so to select $\mathbf{v}_{\mathrm{s}}(t)$ makes possible long trips in short times at high speeds. The times are measured to be the same by travelers inside the sphere and observers outside the sphere. The speeds are those measured by the external observers. The travelers, unless they look at fixed points outside the sphere, will be unaware that they are moving, for everything inside, light included, behaves as if the sphere were at rest.

This somewhat counterintuitive motion of the spherical region involves a distortion of space-time highly localized at the region's boundary. As Alcubierre noted, a mechanism for producing that distortion, however it might be designed, would fit well the picturesque name 'warp drive' familiar from science fiction. In this paper I shall show that such a warp drive can be made to serve as an antigravity device, and shall describe a topological design that causes the idea of constructing one to seem a little less far-fetched than conventional wisdom would suggest.

The space-time metric that Alcubierre exhibited achieves its effect by replacing the zero velocity of the motionless points of empty space by the translational velocity $\mathbf{v}_{\mathrm{s}}(t)$, but only (to a near approximation) inside a sphere of radius R, which sphere we may for purposes of the present discussion take to be the skin of a spaceship propelled by the warp drive (with us in it, let us say). This sphere centers on the point at $\mathbf{x}_{\mathrm{s}}(t)$, which moves so that $d\,[\mathbf{x}_{\mathrm{s}}(t)]/dt = \mathbf{v}_{\mathrm{s}}(t)$ at all times. The restriction of the motion to the x direction, introduced for simplicity's sake, may be dispensed with. The space-time produced by this distortion of flat Minkowski space-time has then the proper-time line element $d\tau$ given by

$$d\tau^2 = dt^2 - |d\mathbf{x} - \mathbf{u}(t,\mathbf{x})\,dt|^2, \tag{1}$$

where $\mathbf{u}(t,\mathbf{x}) := \mathbf{v}_{\mathrm{s}}(t) f(r_{\mathrm{s}}(t,\mathbf{x}))$ and $r_{\mathrm{s}}(t,\mathbf{x}) := |\mathbf{x} - \mathbf{x}_{\mathrm{s}}(t)|$, the function f being defined by

$$f(r) := \frac{\tanh(\sigma(r+R)) - \tanh(\sigma(r-R))}{2\tanh(\sigma R)}, \tag{2}$$

so that, as $\sigma \to \infty$, $f(r)$ tends to 1 if $|r| < R$ but to 0 if $|r| > R$. Every space-time path with 4-velocity $[\![\, 1, \mathbf{u} \,]\!]$, thus with 3-velocity $d\mathbf{x}/dt = \mathbf{u}$, is geodesic; outside the spaceship $\mathbf{u} \approx \mathbf{0}$, inside $\mathbf{u} \approx \mathbf{v}_{\mathrm{s}}$ ($= \mathbf{v}_{\mathrm{s}}$ at the center). The function f interpolates between the exterior velocity $\mathbf{0}$ and the interior velocity $\mathbf{v}_{\mathrm{s}}$, abruptly replacing the one with the other at the spaceship's skinny boundary, where $r_{\mathrm{s}}(t,\mathbf{x}) = R$.

If the points of space are themselves not sitting still, rather are streaming along with flow velocity $\mathbf{v}(t,\mathbf{x})$, the same velocity interpolation takes the form

$$\mathbf{u}(t,\mathbf{x}) := \mathbf{v}_{\mathrm{a}}(t,\mathbf{x})[1-f(r_{\mathrm{s}}(t,\mathbf{x}))] + \mathbf{v}_{\mathrm{s}}(t)f(r_{\mathrm{s}}(t,\mathbf{x})), \tag{3}$$

with $\mathbf{v}_{\mathrm{a}}$, the ambient velocity, equal to $\mathbf{v}$. The velocity field $[\![1,\mathbf{u}]\!]$ remains geodesic, but now outside the spaceship $\mathbf{u}\approx\mathbf{v}_{\mathrm{a}}$, while inside still $\mathbf{u}\approx\mathbf{v}_{\mathrm{s}}$. This would be the situation if the spaceship were immersed in a gravitational field representable by a metric

$$dt^2 - |d\mathbf{x} - \mathbf{v}(t,\mathbf{x})\,dt|^2, \tag{4}$$

because for this metric the geodesic 4-velocity $[\![1,\mathbf{v}]\!]$ can be interpreted as that of a point of space moving with 3-velocity $\mathbf{v}$ (with respect to an immobile background space, one has to say). Using our warp drive to distort this metric to that of equation (1), we can, with the choice of $\mathbf{v}_{\mathrm{s}}$ at our disposal, navigate freely in the gravitational field, even stop at will to inspect our environs. The Schwarzschild field has such a representation, for the Schwarzschild metric of an object of active gravitational mass m, namely

$$(1-2m/\rho)\,dT^2 - (1-2m/\rho)^{-1}d\rho^2 - \rho^2 d\vartheta^2 - \rho^2(\sin\vartheta)^2 d\varphi^2, \tag{5}$$

is brought by the transformation $T = t - \int\sqrt{2m/\rho}\,(1-2m/\rho)^{-1}\,d\rho$ to the form

$$dt^2 - (d\rho + \sqrt{2m/\rho}\,dt)^2 - \rho^2 d\vartheta^2 - \rho^2(\sin\vartheta)^2 d\varphi^2, \tag{6}$$

and then, upon conversion of the spherical coordinates $[\![\rho,\vartheta,\varphi]\!]$ to cartesian, to the form (4) with $\mathbf{v}(t,\mathbf{x}) = \mathbf{v}_{\mathrm{Sch}}(t,\mathbf{x}) := -\sqrt{2m/|\mathbf{x}|}\,(\mathbf{x}/|\mathbf{x}|)$. In this representation the acceleration of a radially moving test particle is $-\nabla(-\frac{1}{2}|\mathbf{v}_{\mathrm{Sch}}|^2)$ $(=-m/|\mathbf{x}|^2)$, so $-\frac{1}{2}|\mathbf{v}_{\mathrm{Sch}}|^2$ plays the role of Newtonian gravitational potential.

With the ambient gravitational field thus canceled inside the spaceship we normally will float about, bouncing off the bulkheads. When the novelty of this wears off, we can gain the illusion of *terra firma* under our feet by simulating the presence of Earth beneath the spaceship. All that is necessary is to modify the interior velocity to $\mathbf{v}_{\mathrm{s}}(t)+\mathbf{v}_g(t,\mathbf{x})$, with $\mathbf{v}_g$ defined by

$$\mathbf{v}_g(t,\mathbf{x}) := -\sqrt{2g(R-(\mathbf{x}-\mathbf{x}_{\mathrm{s}}(t))\cdot\mathbf{n})}\;\mathbf{n}, \tag{7}$$

in which $\mathbf{n}$, the 'upward pointing' unit vector normal to the spaceship's deck, is presumed to have a fixed direction. That will give everything inside the spaceship and 'above' the deck an acceleration $-g\mathbf{n}$ toward the deck. This acceleration, attributable to the spatial non-uniformity of $\mathbf{v}_g(t,\mathbf{x})$, will be with respect to the internal space of the spaceship, unlike $d[\mathbf{v}_{\mathrm{s}}(t)]/dt$, which, being spatially uniform inside and not in effect outside, is an acceleration with respect only to the space outside the ship and therefore goes unnoticed within.

If the Schwarzschild object is a black hole, we can with our warp drive go below its horizon at $\rho=2m$ and come back up unscathed. The drive has only to work a little harder just below the horizon, where $|\mathbf{v}_{\mathrm{a}}| = |\mathbf{v}_{\mathrm{Sch}}| > 1$, than just above, where $|\mathbf{v}_{\mathrm{a}}| = |\mathbf{v}_{\mathrm{Sch}}| < 1$. To escape from under the horizon we need only command the drive to give $\mathbf{v}_{\mathrm{s}}(t)$ a non-zero outward component. Nor will it matter that the black hole is rotating. From its expression in Boyer-Lindquist 'Schwarzschild-like' coordinates $[\![T,\rho,\vartheta,\varphi]\!]$ [2] the Kerr rotating black hole metric transforms to

$$\begin{aligned} dt^2 &- \left[1 - a^2(\sin\vartheta)^2\frac{\Sigma\Delta-(2m\rho)^2}{\Sigma\Delta^2}\right]\left(d\rho + \frac{\sqrt{2m\rho\,(\rho^2+a^2)}}{\Sigma}\,dt\right)^2 - \Sigma\,d\vartheta^2 \\ &- \left[\frac{4am\rho\,(\sin\vartheta)^2\sqrt{2m\rho\,(\rho^2+a^2)}}{\Sigma\Delta}\right]\left(d\rho + \frac{\sqrt{2m\rho\,(\rho^2+a^2)}}{\Sigma}\,dt\right)\left(d\varphi - \frac{2am\rho}{\Sigma\Delta}\,dt\right) \\ &- \left[(\rho^2+a^2)(\sin\vartheta)^2 + \frac{2m\rho}{\Sigma}\,a^2(\sin\vartheta)^4\right]\left(d\varphi - \frac{2am\rho}{\Sigma\Delta}\,dt\right)^2, \end{aligned} \tag{8}$$

with $\Sigma = \rho^2 + a^2(\cos\vartheta)^2$ and $\Delta = \rho^2 - 2m\rho + a^2$, when the substitution $T = t - \int \sqrt{2m\rho\,(\rho^2 + a^2)}\ \Delta^{-1}\ d\rho$ is made [3]. This has the form

$$dt^2 - \gamma_{ij}(x)(dx^i - v^i(x)\,dt)(dx^j - v^j(x)\,dt), \tag{9}$$

with $[\![\, x^i \,]\!] = [\![\, \rho, \vartheta, \varphi \,]\!]$, $v^\rho = -\sqrt{2m\rho\,(\rho^2 + a^2)}\big/\Sigma$, $v^\vartheta = 0$, and $v^\varphi = 2am\rho/\Sigma\Delta$. Although the spatial geometry described by the metric γ_{ij} is not flat if $a \neq 0$, velocity interpolation like that above will replace the velocity v by a velocity v_s inside the spherical spaceship, the result being a metric

$$dt^2 - \gamma_{ij}(x)(dx^i - u^i(t,x)\,dt)(dx^j - u^j(t,x)\,dt), \tag{10}$$

where

$$u^i(t,x) := v_\mathrm{a}{}^i(x)[1 - f(r_\mathrm{s}(t,x))] + v_\mathrm{s}{}^i(t) f(r_\mathrm{s}(t,x)), \tag{11}$$

$v_\mathrm{a} = v$, and $r_\mathrm{s}(t,x)$ is the geodesic distance from the center of the sphere at $x_\mathrm{s}(t)$ to the point at x, measured by the metric γ_{ij}. As in the euclidean case, the velocity field $\partial_t + u^i\partial_i$ is geodesic, so the points of space inside the ship will move in concert almost as a rigid body with velocity v_s.

For the non-rotating black hole the volume expansion θ at time t of the geodesic velocity field $\partial_t + u^i\partial_i$ is just the flat-space divergence of $\mathbf{u}$, calculated from equation (3) with $\mathbf{v}_\mathrm{a} = \mathbf{v}_\mathrm{Sch}$. Figure 1 is a shaded contour plot of θ restricted to an arbitrary plane through the x axis, with $\sigma = 8$, $R = 1$, $m = 5$, $\mathbf{x}_\mathrm{s}(t) = [\![\, 4, 0, 0 \,]\!]$, and $\mathbf{v}_\mathrm{s}(t) = \mathbf{0}$. Lowest (negative) values of θ show as black, highest (positive) values as white. The spaceship is holding its position with $|\mathbf{x}| = 4$, well inside the black hole's event horizon at $|\mathbf{x}| = 2m = 10$. Figure 2 is like Figure 1, but with the plane fixed as the xy plane and $\mathbf{v}_\mathrm{s}(t) = [\![\, 0, 2, 0 \,]\!]$. The spaceship is at 'periholion', passing the black hole below the horizon along a path (vertical in the figure) that is tangential to the sphere of symmetry at $|\mathbf{x}| = 4$. These plots are analogs of the surface plot of θ shown in Figure 1 of Alcubierre's letter.

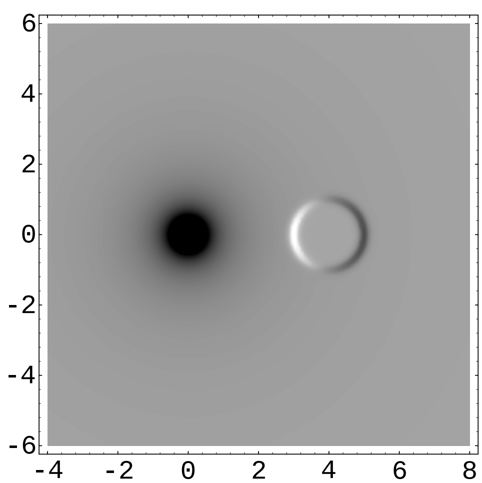

Figure 1. Spaceship at rest near a black hole.

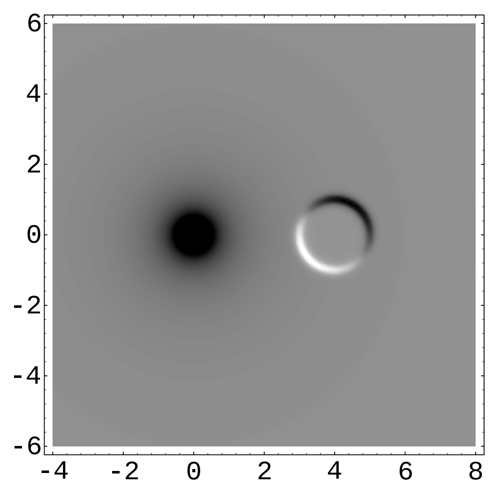

Figure 2. Spaceship passing near the black hole.

Alcubierre reasoned that his spaceship moves along with velocity $\mathbf{v}_\mathrm{s}$ because some unspecified mechanism involving 'exotic' matter continuously shrinks space in front of it (where $\theta < 0$) and expands space behind it (where $\theta > 0$). That interpretation extended to the current development would say that the black and the white regions of the contour plots are places where space is shrinking (black) or expanding (white). An alternative way of describing what is happening arises from an idea in [4]. In that paper I attributed the gravity of the Schwarzschild field to the "internal, relative motions" of a "more or less substantial 'ether', pervading all of space-time". The velocity $\mathbf{v}_\mathrm{e}$ of this 'ether' is just the velocity $\mathbf{v}_\mathrm{Sch}$ given above, which one recognizes as the velocity of an observer falling freely from rest at $\mathbf{x} = \infty$. The role of gravitational potential thus falls to the scalar field $-\frac{1}{2}|\mathbf{v}_\mathrm{e}|^2$, the negative of the 'specific kinetic energy' of the ether.

Application of this 'ether flow' picture to the present discussion would suggest that the black regions in Figure 1 indicate the presence of ether sinks, one at the Schwarzschild singularity and one spread over the upper and some of the lower hemisphere of the spaceship's skin, whereas the white region indicates an ether source spread over the remainder of the lower hemisphere of the skin. The enveloping of the spaceship in this way by an ether sink and an ether source of just the right strengths allows the ether inside the spaceship to remain at rest, aloof from the headlong rush of the surrounding ether into the sink at $\mathbf{x} = \mathbf{0}$. Modification of the strengths and locations of the enveloping sink and source as in Figure 2 produces an outward velocity of the sink, the source, and the ether inside the spaceship, thus permits the ship to escape the black hole. Far away from gravitating matter, as in Alcubierre's example, $\mathbf{v}_{\mathrm{e}} \approx \mathbf{0}$. The ether outside the spaceship is nearly at rest, but that ahead is drawn continuously into a sink region at the front of the spaceship at a rate determined by the forward speed of the ship, while that behind is added to at a similar rate out of a source region in back. The ether inside the spaceship moves along in step with the sink and the source. Having no internal, relative motion, it produces no gravity.

Just what the 'ether' might be is a question not addressed in [4]. Whether one thinks of the ether as a fluid of some kind spread throughout space, or as space itself flowing in time, or as just a convenient fiction is largely a matter of taste — the mathematics is the same in every case. The idea of *something* flowing does, however, suggest a way for the warp drive to be brought into existence. Creation of a $\theta < 0$ and a $\theta > 0$ region independent of one another, enveloping the spaceship and able to drive it, would require accumulation of a considerable amount of attractive active gravitational mass in front of the ship and a hard-to-imagine independent accumulation of a similar amount of repulsive active gravitational mass behind it. But should these regions be an ether source and an ether sink, connected so that the ether disappearing into the sink reappears at the source in the manner that a river disappears underground only to spring forth at a point downstream, then perhaps they can be created more easily.

In the ether flow paper, to remedy the undesirable destruction of ether by the singularity at the black hole sink, I proposed an alternative to the Schwarzschild black hole, and termed it a 'drainhole'. The drainhole is a static solution of the usual coupled Einstein–scalar-field equations, but with non-standard coupling polarity. Its metric, dependent on a mass parameter m and a parameter $n > |m|$, takes the spherically symmetric, radial ether flow form, analogous to the form (6) of the Schwarzschild metric in spherical coordinates,

$$dt^2 - (d\rho - v_{\mathrm{e}}^{\rho}(\rho)dt)^2 - r^2(\rho)d\vartheta^2 - r^2(\rho)(\sin\vartheta)^2 d\varphi^2, \tag{12}$$

where

$$r(\rho) = \sqrt{\rho^2 - 2m\rho + n^2}\,\exp\left(\frac{m}{n}\alpha(\rho)\right) \tag{13}$$

and

$$v_{\mathrm{e}}^{\rho}(\rho) = -\mathrm{sgn}(m)\left[1 - \exp\left(-\frac{2m}{n}\alpha(\rho)\right)\right]^{1/2}, \tag{14}$$

with

$$\alpha(\rho) = \frac{n}{\sqrt{n^2 - m^2}}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{\rho - m}{\sqrt{n^2 - m^2}}\right)\right] \tag{15}$$

(ρ here corresponds to $\rho + m$ in [4], and ranges from $-\infty$ to ∞). If $m > 0$, then $r(\rho) \sim \rho$ and $v_{\mathrm{e}}^{\rho}(\rho) \sim -\sqrt{2m/\rho}$, as $\rho \to \infty$, so the drainhole's behavior is asymptotic to that of the black hole as $\rho \to \infty$. Unlike the black hole, however, the drainhole is geodesically complete. Where the black hole has two asymptotically flat outer regions, connected for a short time by a 'throat' at the horizon, and two inner regions, each with a central singularity where curvatures become infinite, the drainhole has only two asymptotically flat regions (one where $\rho \to \infty$, the other where $\rho \to -\infty$) connected by a throat for as long as the drainhole exists, and has no singularity or horizon at all.

In the black hole the ether flow acceleration is everywhere inward, so the black hole is gravitationally attractive on both sides. Contrarily, the Schwarzschild white hole, whose metric is given by (5) with $m < 0$, can only repel gravitationally. It consequently is not susceptible to an ether flow description, there being no possibility of an observer's falling freely *from* (or *to*) *rest* at $|\mathbf{x}| = \infty$, and therefore no vector field corresponding to the $\mathbf{v}_{\mathrm{Sch}}$ of the black hole. Nevertheless, accumulation of small black holes in front of the spaceship and small white holes, if such could be produced, behind it would create a warp drive consisting of multiple sinks in front, taking in ether with no place to put it, and unrelated multiple repellers in back, somehow fabricating new space out of nothing and pushing it away by a mechanism unknown.

A drainhole can be thought of as a happy union of a black hole and a white hole in which the (w)hole is better than the sum of its parts. The acceleration of the radially flowing ether in a drainhole is given by $d^2\rho/dt^2 = d\,[v_e^\rho(\rho)]/dt = -m/r^2(\rho)$. The drainholes with $m < 0$ are metrically indistinguishable from those with $m > 0$. In each the ether flows from one asymptotically flat region, through the throat, and out into the other, accelerating at every point in the direction of the flow, which is inward (toward the throat) on the 'high' side, where $\rho/2m > 1$, and outward (away from the throat) on the 'low' side, where $\rho/2m < 1$. The ether comes out faster than it went in, and flows the faster the farther out it travels. Thus the drainhole appears on the high side as a gravitationally attracting ether sink, and on the low side as a gravitationally repelling ether source. Not only that, but the strength of the repulsion exceeds that of the attraction, by a percentage calculable as approximately $\pi m/n$ if $0 < m \ll n$. What is more, to keep the drainhole throat open does not require that the ether flow rapidly, or even at all — the throat's smallest possible constriction, which occurs when the ether is not flowing, that is, when $m = 0$ so that $v_e^\rho = 0$, is a two-sphere of area $4\pi n^2$. All of this is established in [4].

Drainholes allowed to evolve can appear and disappear. In [5] I derived a solution of the coupled Einstein–scalar-field equations in the form of a space-time manifold $\mathcal{M}_a$ comprising, if the parameter $a \neq 0$, two asymptotically flat regions connected by a throat that constricts to a point and immediately reopens and begins to enlarge (a phenomenon prefigurative of the 'scalar field collapse' studied later in [6–10]). The metric is $dt^2 - d\rho^2 - r^2(t,\rho)d\vartheta^2 - r^2(t,\rho)(\sin\vartheta)^2 d\varphi^2$, with $r^2(t,\rho) = a^2t^2 + (1 + a^2)\rho^2$. Combining the $t > \rho$ region of $\mathcal{M}_a$ for $a \neq 0$ and the $t < \rho$ region of $\mathcal{M}_0$ produces a drainhole in which the throat is absent when $t \leq 0$, but present when $t > 0$. Combining the $t < \rho$ region of $\mathcal{M}_a$ and the $t > \rho$ region of $\mathcal{M}_0$ produces a drainhole in which the throat is present when $t < 0$, but absent when $t \geq 0$. The ether is at rest, but the existence of analogous solutions with the ether flowing is plausible.

If small wormhole-like versions of these drainholes could be manufactured in great numbers, with their high sides distributed over one face of a closed vessel of spherical (or perhaps spheroidal!) shape, and their low sides spread over the opposite face, the result would be an ether by-pass of the vessel consisting of a multitude of tiny topological tunnels. With such an ether by-pass the vessel could be shielded from the external gravity embodied in the flow of the ether, and could become a gravity defying spaceship — in short, a warp drive would exist. It is easy enough to imagine the mathematical existence of such a diverted-ether-flow space-time configuration (imagine the spatial topology with the velocity field of the flow painted on), but concrete physical existence is another matter. One is tempted to speculate that something as conceptually simple as a generator of particle-antiparticle pairs, coupled to an accelerator to separate the particles and the antiparticles and spread them over opposite faces of the vessel, would do the job. This, though, presumes more than is known by Earthlings about the active gravitational masses of such things as electrons and positrons.

Whatever the means by which such a warp drive might be realized — if realized one ever should be — it is worthy of note that once in existence the drive would be able to levitate a heavily loaded vessel almost as easily as it would the vessel alone. To levitate an empty vessel on Earth, whose active gravitational mass M_{E} is treated as concentrated at its center a distance R_{E} below its surface, the drive must merely provide a by-pass for ether flowing downward with velocity $\sqrt{2M_{\mathrm{E}}/R_{\mathrm{E}}}$, which is Earth's escape velocity of about 1.1×10^4 m s^{-1}, and acceleration $M_{\mathrm{E}}/R_{\mathrm{E}}^2$, Earth's surface gravitational acceleration of about 9.8 m s^{-2}. A cargo weighing many tons would constitute an ether sink inside the spaceship, but one whose gravitational escape velocity would be negligible in comparison to Earth's. It would increase the ether velocity and acceleration above the vessel by relatively insignificant amounts, and would decrease them by similar amounts below the vessel. The change in burden on the drive would be slight. On the other hand, to progress from supporting the vessel against the Earth's gravitational pull to zooming it along through space at the speed of light could require a considerable increase in the drive's efficiency. Its tiny topological tunnels would be required to pass ether at 3.0×10^8 m s^{-1}, instead of a paltry 1.1×10^4 m s^{-1}. Maintaining the velocity through the tunnels at the speed of light, once attained, could be no more taxing than allowing a flowing river to keep on flowing. Attaining that velocity in a reasonable time would present the difficulty. To go from rest to traveling at lightspeed in an hour, a day, or a year would, according to the time-honored formula $v = at$, demand that the ether be constantly accelerated through the tunnels at about 8.3×10^4 m s^{-2}, 3.5×10^3 m s^{-2}, or 9.5 m s^{-2}, respectively. The ether acceleration just sufficient for levitation at Earth's surface thus would produce lightspeed only after about one year. This acceleration, if sustained for 50 light-years and reversed for another 50, would cause a one-way trip of 100 light-years to last 20 ($= 2\sqrt{2 \cdot 50}$) years. If lightspeed could be attained in a day, the time required would be $20/\sqrt{365}$ (≈ 1.05) years; if in an hour, $20/\sqrt{8760}$ years (≈ 78 days).

The maximum acceleration of the ether through a static drainhole occurs where the throat is most constricted, that is, at $\rho = 2m$, where r has its minimum value. The magnitude of the acceleration there is $|m|/r^2(2m)$, a number of the order of $|m|/n^2$ whatever the relative sizes of m and n. To put a face on this number, suppose $m = m_{\text{neutron}} \approx 1.2 \times 10^{-54}$ m (the rest mass — and *presumably* a close approximation to the active gravitational mass — of the neutron in units in which $G = c = 1$, which have been assumed here) and $n = n_{\text{Pla}} \approx 1.6 \times 10^{-35}$ m (the Planck distance). From (12) one finds that for light rays $(d\rho/dt - v_e^\rho(\rho))^2 + r^2(\rho)(d\vartheta/dt)^2 + r^2(\rho)(\sin\vartheta)^2(d\varphi/dt)^2 = 1$, so that the speed of light with respect to the ether, thus with respect to an observer in radial free fall through the drainhole, is indeed 1. This makes 1 m $\approx 3.3 \times 10^{-9}$ s, so that $n_{\text{Pla}} \approx 5.3 \times 10^{-44}$ s, and therefore $m/n^2 \approx 4.3 \times 10^{32}$ m s^{-2}. If, on the other hand, $n = n_{\text{electron}} \approx 2.8 \times 10^{-15}$ m (the classical radius of the electron), then $m/n^2 \approx 1.3 \times 10^{-8}$ m s^{-2}. Thus tiny topological tunnels of radius n_{Pla} would likely produce a far stronger drive than tunnels of radius n_{electron}, *provided* they could be created and distributed in quantities sufficient to divert all of the ether flowing into the spaceship's hull.

At this point we are off the end of the good highway built on firm mathematics, and at risk of wandering lost in the desert. We had best retreat to the pavement and see what can be done to extend it beyond the horizon. Perhaps we shall be able only to survey a mathematical route on which we can never pour the concrete of physical existence. Even so, a warp drive would be such a marvelous thing to possess that one cannot help longing for its creation. Should we not set our minds to the task? And if we are not alone in the Universe, is it not likely that others already have set theirs, perhaps to good effect?

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Homer G. Ellis
Department of Mathematics
University of Colorado at Boulder
395 UCB
Boulder, Colorado 80309-0395

Telephone: (303) 492-7754 (office); (303) 499-4027 (home)
Email: ellis@euclid.colorado.edu
Fax: (303) 492-7707