Quantum effects from a purely geometrical relativity theory[1]

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Abstract. A purely geometrical relativity theory results from a construction that produces from three-dimensional space a happy unification of Kaluza's five-dimensional theory and Weyl's conformal theory. The theory can provide geometrical explanations for the following observed phenomena, among others: (a) lifetimes of elementary particles of lengths inversely proportional to their rest masses; (b) the equality of charge magnitude among all charged particles interacting at an event; (c) the propensity of electrons in atoms to be seen in discretely spaced orbits; and (d) 'quantum jumps' between those orbits. This suggests the possibility that the theory can provide a deterministic underpinning of quantum mechanics like that provided to thermodynamics by the molecular theory of gases.

This presentation is intended to show that some of the phenomena thought to be explainable only through the process of 'quantizing' a classical relativistic theory can be explained, qualitatively and to some degree quantitatively, by a purely geometrical relativity theory based on and derived solely from the geometry of three-dimensional space.[2] A simple geometric construction applied iteratively generates new dimensions beyond the basic three of space. The first application produces space-time, the second produces space-time–time, and so on. Space-time is a generalized de Sitter space. Space-time–time, in which quantum effects show up, is a happy hybrid of two notable attempts at a unified theory of gravity and electromagnetism: the Kaluza five-dimensional geometry [3] and the Weyl conformal geometry [4]. Brought together in this way those theories lose their undesirable properties while retaining their useful ones. That the space-time–time geometry both induces quantum effects and includes gravity (along with other fields) calls into question the rationale behind the search for a quantum theory of gravity. This presentation and its author might therefore be regarded as intruders from a school devoted to "Escapes from Quantum Gravity", conducted in a parallel universe.

The geometric construction in question can be understood initially by reference to Fig. 1, which shows in cross section two neighboring spheres S and S' in euclidean 3-space $\mathbb{E}^3$ with centers C and C' the distance ds apart, and with radii R and $R + dR$. The angle $d\alpha$ in which they intersect is found, by application of Pythagoras' theorem to the infinitesimal right triangle in the middle, to be given by $d\alpha^2 = (1/R^2)(ds^2 - dR^2)$, which shows $d\alpha$ to be the line element of a metric of diagonal signature $+ + + -$ on the four-dimensional manifold $\mathcal{M}^4$ whose points

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[2] An early, imperfect description of the theory can be found in [1], later, detailed descriptions in [2].

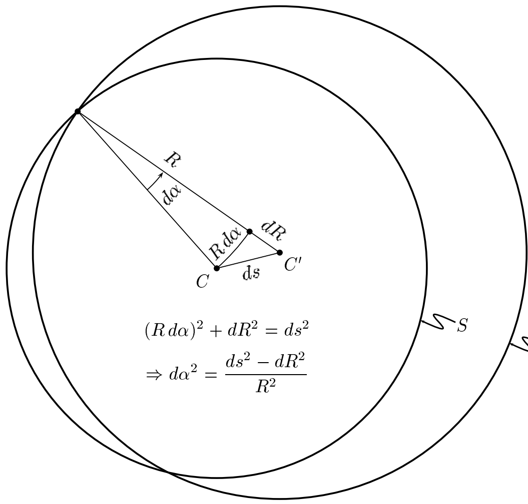

Figure 1. The ‘angle’ line element.

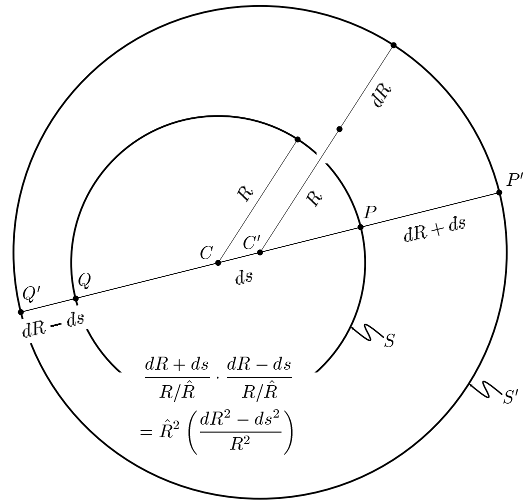

Figure 2. The ‘displacement’ line element.

are the 2-spheres of $\mathbb{E}^3$. Note that $d\alpha$ is invariant under conformal transformations of $\mathbb{E}^3$.

A more general version of the construction is exhibited in Fig. 2. Here, instead of $\mathbb{E}^3$ there is a three-dimensional manifold $\mathcal{M}^3$ with a positive-definite riemannian metric G, whose geodesic 2-spheres are the points of $\mathcal{M}^4$; S and S' are two such spheres, $Q'QCC'PP'$ is the geodesic through their centers C and C', and the line element is generated as the product of the distances (relative to a scaled radius $R/\hat{R}$) by which P and Q are displaced when S is magnified radially by the factor $1 + dR/R$ and its center C is shifted a distance ds along the geodesic to C', which in effect converts S to S'. The line element that results is given by $d\tau^2 = (\hat{R}/R)^2(dR^2 - ds^2)$.

Defining a new coordinate t by $t := -\ln(R/\hat{R})$ makes

$$d\tau^2 = \hat{R}^2 dt^2 - e^{2t} ds^2, \tag{1}$$

a generalization of the de Sitter space-time metric for an empty expanding universe of uniform radius of curvature $\hat{R}$, to which metric it reduces when $\mathcal{M}^3 = \mathbb{E}^3$. Thus, by means of a simple geometric construction we have produced from (more aptly, discovered within) the geometry of three-dimensional space the geometry of a four-dimensional space-time.

The tensor product version of the metric specified by equation (1) is

$$\hat{G} = \hat{R}^2(dt \otimes dt) - e^{2t} G = \hat{R}^2(dt \otimes dt) - e^{2t}(dx^m \otimes g_{mn} dx^n), \tag{2}$$

where G is the metric of $\mathcal{M}^3$. For this space-time metric ∂_t is a ‘conformal semi-Killing’ vector field, in the sense that $\mathcal{L}_{\partial_t}\hat{G} = -2\,e^{2t}G$, where $\mathcal{L}_{\partial_t}$ denotes Lie differentiation along ∂_t. Observing that $-e^{2t}G = \hat{G} - (\hat{G}\partial_t\partial_t)^{-1}(\hat{G}\partial_t \otimes \hat{G}\partial_t)$, one sees that to capture least restrictively in a generic space-time metric $\hat{G}$ on a manifold $\mathcal{M}^4$ the essence of the geometrical construction in question it is sufficient to subject $\hat{G}$ to the constraint that there exist on $\mathcal{M}^4$ a time-like vector field ξ such that $\mathcal{L}_\xi \hat{G} = 2\,[\hat{G} - (\hat{G}\xi\xi)^{-1}(\hat{G}\xi \otimes \hat{G}\xi)]$. It is then easy to see that, in a coordinate system $[\![\, x^m, t \,]\!]$ adapted to ξ so that $\xi = \partial_t$, $\hat{G}$ takes the form

$$\begin{aligned} \hat{G} &= \phi^2(A + dt) \otimes (A + dt) - e^{2t} G \\ &= \phi^2(A_m dx^m + dt) \otimes (A_n dx^n + dt) - e^{2t}(dx^m \otimes g_{mn} dx^n), \end{aligned} \tag{3}$$

with ϕ, A_m, and g_{mn} independent of t.

Having arrived at the space-time geometry described by the metric $\hat{G}$ on the manifold $\mathcal{M}^4$, we can repeat the construction, applying it this time to the geodesic 3-spheres of $\mathcal{M}^4$ to produce a metric on the five-dimensional manifold $\mathcal{M}^5$ whose points are those geodesic 3-spheres. Generalizing that metric in the manner that produced (3) we obtain

$$\begin{aligned}\hat{G} &= e^{2\zeta}G + \hat{\epsilon}\phi^2(A+d\zeta)\otimes(A+d\zeta)\\ &= e^{2\zeta}(dx^\mu\otimes g_{\mu\nu}\,dx^\nu) + \hat{\epsilon}\phi^2(A_\mu\,dx^\mu + d\zeta)\otimes(A_\nu\,dx^\nu+d\zeta),\end{aligned} \tag{4}$$

referred to a coordinate system $[\![\, x^\mu, \zeta \,]\!]$ such that $\xi = \partial_\zeta$, where now G is the space-time metric and ϕ, A_μ, and $g_{\mu\nu}$, the counterparts of the previous ϕ, A_m, and g_{mn}, are independent of ζ. The factor $\hat{\epsilon}$ enters because there are two kinds of spheres (hyper-hyperboloids, actually) in space-time. Those whose points lie in spacelike directions from their centers require that $\hat{\epsilon}=1$, those whose points lie in timelike directions from their centers require $\hat{\epsilon}=-1$. In either case, because the construction that produced $\hat{G}$ from space-time is the same one that produced space-time from space, it is justified, indeed unavoidable, to label the manifold $\mathcal{M}^5$ with the metric $\hat{G}$ a space-time–time, and to identify ζ as a temporal coordinate, albeit of a secondary nature distinct from that of the primary time coordinate t. Ultimately the $\hat{\epsilon}=\pm1$ distinction should be resolved by allowing ζ to be a complex coordinate. For present purposes, however, ζ will be kept real and $\hat{\epsilon}$ will be 1, the points of $\mathcal{M}^5$ being therefore the spacelike spheres of $\mathcal{M}^4$.

The geometry of space-time–time is the previously mentioned happy hybrid of the Kaluza and the Weyl geometries. As in Kaluza's theory, A is the space-time covector potential of the electromagnetic field 2-form F defined by $F := -2\,d_\wedge A$. The coordinate transformation $\zeta' = \zeta - \lambda$, with λ independent of ζ, generates in one stroke both the electromagnetic gauge transformation $A' = A + d\lambda$ and the Weyl conformal transformation $G' = e^{2\lambda}G$. The geodesic paths of $\hat{G}$ are taken to be the histories of test particles. To analyze these histories for physical content one introduces the frame system $\{e_\mu, e_5\} := \{\partial_\mu - A_\mu\,\partial_\zeta, \phi^{-1}\partial_\zeta\}$ and its dual coframe system $\{\omega^\mu,\omega^5\} := \{dx^\mu, \phi(A_\mu\,dx^\mu + d\zeta)\}$, for which $\hat{G} = e^{2\zeta}(\omega^\mu\otimes g_{\mu\nu}\,\omega^\nu) + \omega^5\otimes\omega^5$ and e_5 is orthogonal to the e_μ. The velocity $\dot{p}$ of a path $p\colon\mathbb{R}\to\mathcal{M}^5$ then is expressed by $\dot{p} = \dot{p}^\mu e_\mu + \dot{p}^5 e_5$, and one can introduce the following definitions for a test particle whose history is p:

$$P := \hat{G}\dot{p} = (e^{2\zeta}\dot{p}^\kappa g_{\kappa\mu})\omega^\mu + \dot{p}^5\omega^5 =: P_\mu\omega^\mu + P_5\omega^5 \qquad \text{(momentum)}, \tag{5}$$

$$\mathring{m} := (P_\mu\, g^{\mu\nu} P_\nu)^{1/2} = e^{2\zeta}(\dot{p}^\mu g_{\mu\nu}\dot{p}^\nu)^{1/2} \qquad \text{(rest mass)}, \tag{6}$$

$$q := \phi P^5 = \phi\dot{p}^5 = \phi^2(A_\mu\dot{p}^\mu + \dot{\zeta}) \qquad \text{(electric charge)}, \tag{7}$$

$$\tau := \textstyle\int(\dot{p}^\mu g_{\mu\nu}\dot{p}^\nu)^{1/2} = \int e^{-2\zeta}\mathring{m} \qquad \text{(proper time)}, \tag{8}$$

$$u^\mu := \frac{dp^\mu}{d\tau} := \frac{\dot{p}^\mu}{\dot{\tau}} \qquad \text{(proper velocity)}. \tag{9}$$

Of these P and q are gauge-invariant and the others are not, although the condition $\mathring{m}=0$ is gauge-invariant.

Taking p to be a secondarily timelike geodesic parametrized by arclength, that also is primarily timelike in that $\dot{p}^\mu g_{\mu\nu}\dot{p}^\nu > 0$, one has that $|\dot{p}|^2 = \hat{G}(p)\dot{p}\dot{p} = e^{2\zeta}(\dot{p}^\mu g_{\mu\nu}\dot{p}^\nu) + (\dot{p}^5)^2 = e^{-2\zeta}\mathring{m}^2 + (q/\phi)^2 = 1$ and that $\ddot{p} = \ddot{p}^\mu e_\mu + \ddot{p}^5 e_5 = 0$. The equation $\ddot{p}^\mu = 0$ is equivalent to

$$\frac{d(\mathring{m}u^\mu)}{d\tau} + (\mathring{m}u^\kappa)\Gamma_\kappa{}^\mu{}_\lambda u^\lambda = qF^\mu{}_\lambda u^\lambda - \mathring{m}A^\mu + e^{2\zeta}\frac{(q/\phi)^2}{\mathring{m}}(\ln\phi)^{\cdot\mu}, \tag{10}$$

and $\ddot{p}^5 = 0$ is equivalent to

$$\dot{q} = e^{-2\zeta}\mathring{m}^2 = 1-(q/\phi)^2, \qquad \text{also to} \qquad \frac{dq}{d\tau} = \mathring{m} = e^{\zeta}[1-(q/\phi)^2]^{1/2}. \tag{11}$$

Equations (11) and (7), together with $e^{-2\zeta}\mathring{m}^2 + (q/\phi)^2 = 1$, entail that

$$(\mathring{m}^2)^{\cdot} = 2[-\mathring{m}^2 A_\mu + e^{2\zeta}(q/\phi)^2 (\ln\phi)_{.\mu}]\dot{p}^\mu. \tag{12}$$

Equation (10) shows the rate of change of the particle's space-time momentum $\mathring{m}u^\mu$ with respect to its proper time to be governed by four 'forces': the Einstein–Newton force $-(\mathring{m}u^\kappa)\Gamma_\kappa{}^\mu{}_\lambda u^\lambda$, the Lorentz force $qF^\mu{}_\lambda u^\lambda$, the 'Weyl force' $-\mathring{m}A^\mu$, and the 'Kaluza force' $(\mathring{m})^{-1}(q/\phi)^2(\ln\phi)^{\cdot\mu}$. Equation (11) produces generic behavior of q that is exemplified in the solution $q(\hat{\tau}) = \phi\tanh(\hat{\tau}/\phi)$, where $\hat{\tau}$ is secondary proper time and ϕ is taken to be constant. From $e^{-2\zeta}\mathring{m}^2 + (q/\phi)^2 = 1$ follows $\zeta(\hat{\tau}) = \ln(\mathring{m}(\hat{\tau})\cosh(\hat{\tau}/\phi))$. If also $A_\mu = 0$, then equation (12) tells that $\mathring{m}$ is constant, and (8) yields $\tau(\hat{\tau}) = \mathring{m}\int e^{-2\zeta(\hat{\tau})}\,d\hat{\tau} = (\phi/\mathring{m})\tanh(\hat{\tau}/\phi)$, which shows the particle's proper lifetime to be confined to the open interval $(\tau(-\infty), \tau(\infty))$ $(= (-\phi/\mathring{m}, \phi/\mathring{m}))$, thus associates longer lifetimes with smaller rest masses, shorter lifetimes with larger rest masses. Noting in passing the equivalence of inertial mass and passive gravitational mass implied by the two appearances of $\mathring{m}$ in the lefthand member of (10), one divides by $\mathring{m}$ to obtain $du^\mu/d\tau + u^\kappa\Gamma_\kappa{}^\mu{}_\lambda u^\lambda = 0$, which says that the particle's space-time track is a geodesic of the Einstein geometry. The particle's coordinates x^μ will in general have $\hat{\tau}$ dependence similar to that of τ, in consequence of which the particle's space-time track will have an endpoint event $\mathcal{E}_1$ at which it appears suddenly, traveling with velocity $(dx^m/dt)(-\infty)$ (if $dt/d\tau > 0$), and an endpoint event $\mathcal{E}_2$ at which it disappears just as suddenly, traveling with velocity $(dx^m/dt)(\infty)$.

Essential features of this behavior will persist in the generic case where ϕ and A_μ are not restricted. In particular, the space-time track will end at events $\mathcal{E}_1$ and $\mathcal{E}_2$, and q will grow from $-\phi(\mathcal{E}_1)$ to 0 and on to $\phi(\mathcal{E}_2)$ and ζ will depart from and return to ∞, as $\hat{\tau}$ goes from $-\infty$ to ∞. Figure 3 is a schematic representation of a geodesic exhibiting such behavior.

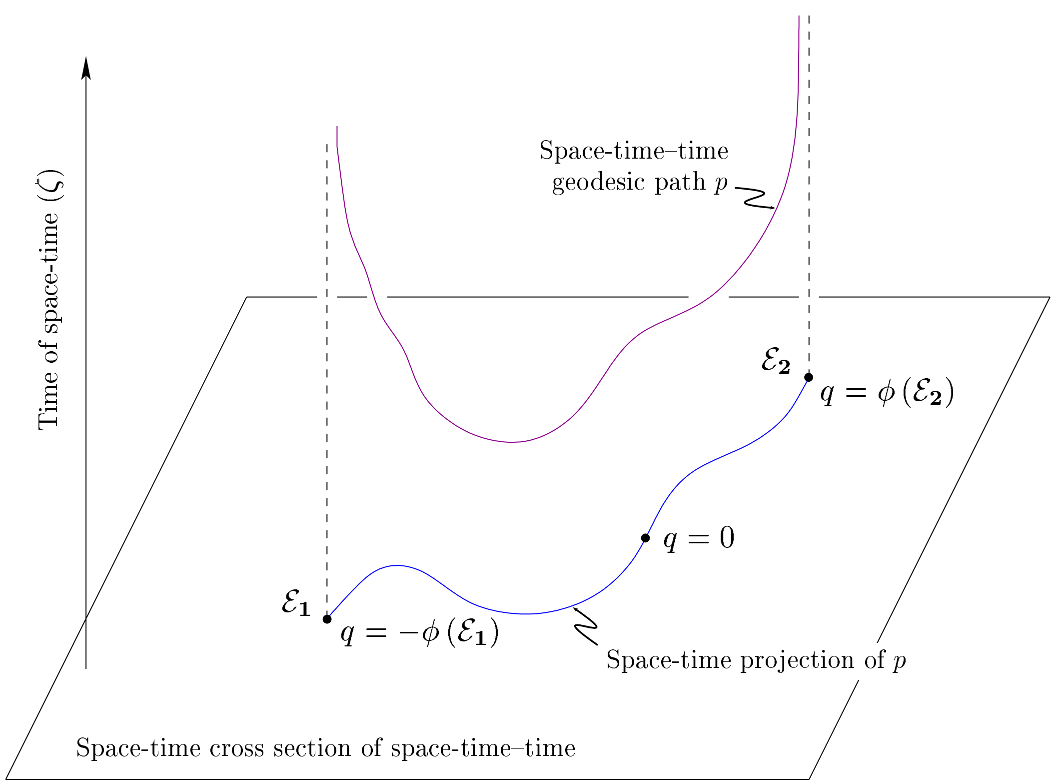

Figure 3. A space-time–time geodesic and its space-time projection.

The variational principle $0 = \delta \int_{\mathcal{D}\times[a,b]} (\hat{\Psi} - \hat{\Psi}_\infty)\, d\hat{V}$, where $\hat{\Psi}$ is the curvature scalar of $\hat{G}$, given by

$$\begin{aligned}\hat{\Psi} &= e^{-2\zeta}\Psi + 6e^{-2\zeta}\left[A^\kappa{}_{:\kappa} - A^\kappa A_\kappa\right] \\ &\quad - 2e^{-2\zeta}\phi^{-1}\left[\phi^{\cdot\kappa}{}_{:\kappa} - 2A^\kappa\phi_{.\kappa}\right] + (1/4)e^{-4\zeta}\phi^2 F_\kappa{}^\lambda F_\lambda{}^\kappa - 20\,\phi^{-2},\end{aligned} \tag{13}$$

$\hat{\Psi}_\infty := \lim_{\zeta\to\infty}\hat{\Psi} = -20\,\phi^{-2}$, and $\mathcal{D}$ is a region of space-time, produces the following field equations, obtained by varying ϕ and A_μ, respectively, where $k := 2(b-a)/(e^{2b} - e^{2a})$:

$$A^\kappa{}_{:\kappa} - 3A^\kappa A_\kappa = -(3/8)k\phi^2 F_\kappa{}^\lambda F_\lambda{}^\kappa - (1/2)\Psi \tag{14}$$

and

$$F^{\mu\kappa}{}_{:\kappa} + 3F^{\mu\kappa}(\ln\phi)_{.\kappa} = -2k^{-1}\phi^{-2}\left[(\ln\phi)^{\cdot\mu} + 6A^\mu\right]. \tag{15}$$

In [5] the analogous equations for space-time are shown to have spherically symmetric solutions of the ‘traversable wormhole’ type, similar to those in [6]. In the space-time–time case, with G describing a nongravitating, static, spherically symmetric, traversable wormhole, with $A = V(r)\,dt$, and with $\ln\phi = U(r)$, numerical integration yields a variety of solutions for which, as $r\to\infty$, $V(r)$ is asymptotic to a Coulomb potential Q/r. For one of these, typical of a large class, $U(r)$ has, for $r \geq 0$, the shape shown in Fig. 4. The spacing of successive bottoms of the potential wells of $U(r)$, located at $r = r_n$, $n = 1, 2, 3, \cdots$, is asymptotic to $2n$, consistent with r_n's growing asymptotically as n^2. Figure 5 is a graph of an artificial version $\bar{U}(r)$ of $\ln\phi$ with $r_n = n^2$ and potential wells of uniform depth, to be used for illustrative purposes.

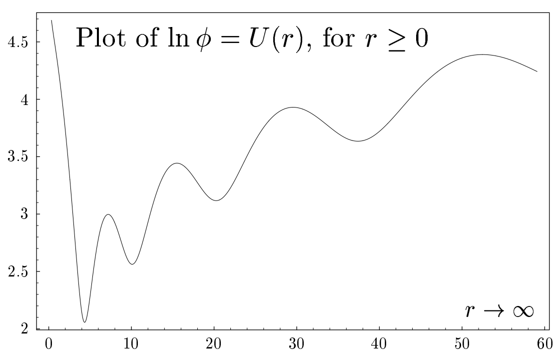

Figure 4. A computed potential.

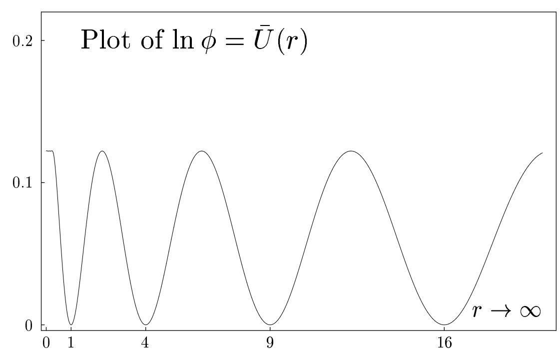

Figure 5. An artificial potential.

Now comes a most remarkable aspect of a test particle's space-time behavior: both as $\hat{\tau} \to -\infty$ and as $\hat{\tau} \to \infty$ the factor $e^{2\zeta}$ that in equation (10) couples the Kaluza force to the momentum rate becomes infinite, which causes that force to *infinitely* dominate the other three, and to push each of the terminal events $\mathcal{E}_1$ and $\mathcal{E}_2$ toward a bottom of one of the potential wells of $\ln\phi$. Thus a scatter plot of $\mathcal{E}_1$ and $\mathcal{E}_2$ generated by random choices of initial conditions $p(0)$ and $\dot{p}(0)$ for the test particle's path p would show high densities near those potential well bottoms, low densities elsewhere. Figure 6 illustrates this behavior, which clearly suggests the possibility of a (for the present, only qualitative) deterministic underpinning of quantum mechanics like that provided to thermodynamics by the molecular theory of gases.

As illustrated in Fig. 3, neither of the events $\mathcal{E}_1$ and $\mathcal{E}_2$ is a projection of a point on the geodesic path p: they are only limits of such projections as $\hat{\tau} \to \pm\infty$ and $\zeta \to \infty$. This suggests that the particle whose space-time track the projection is exists neither at or before $\mathcal{E}_1$ nor at or after $\mathcal{E}_2$, rather exists only between $\mathcal{E}_1$ and $\mathcal{E}_2$. Think back, however, to the original conception of events in space-time as 2-spheres in space, with $t = -\ln(R/\hat{R})$. Under this interpretation the projection of p is a one-parameter family of spheres which converges at either of these events $\mathcal{E}$

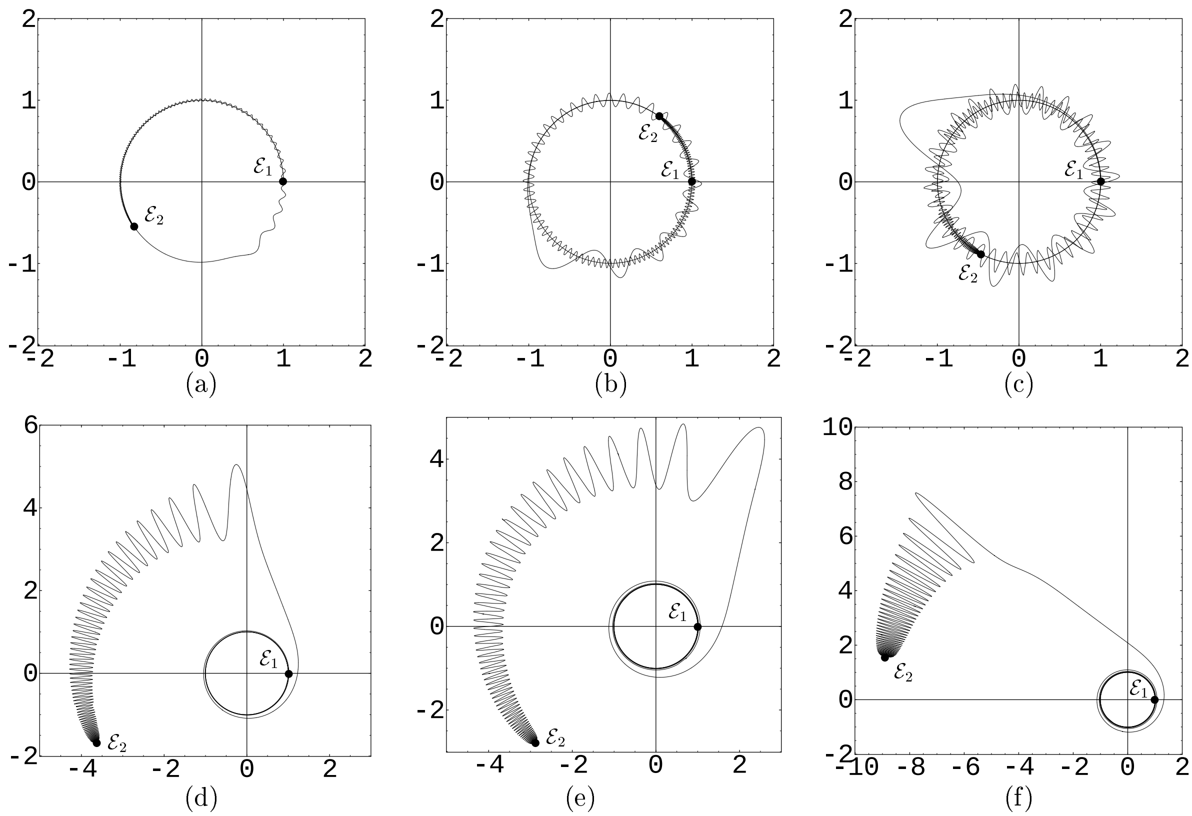

Figure 6. Sample tracks of test particles following space-time–time geodesics governed by the artificial potential $\ln\phi = \bar{U}(r)$ of Fig. 5 and a Coulomb potential $\bar{V}(r) = Q/r$. The particles appear at $\mathcal{E}_1$, follow the track in the counterclockwise direction, and disappear at $\mathcal{E}_2$. In each case $\mathcal{E}_1$ is at $r = 1$, $\varphi = 0$, with $q = -\phi(\mathcal{E}_1) = -1$ and $\mathring{m} = 1$. The initial angular velocities $d\varphi/dt$ are: (a) 0.0050; (b) 0.0101; (c) 0.0151; (d) 0.0352; (e) 0.0502; (f) 0.0602. The numbers of complete revolutions in the orbits, and the locations of $\mathcal{E}_2$, are (a) 1, $r = 0.995545$, $\varphi = 212^\circ$; (b) 3, $r = 0.992591$, $\varphi = 51^\circ$; (c) 4, $r = 0.993775$, $\varphi = 240^\circ$; (d) 5, $r = 4.00752$, $\varphi = 204^\circ$; (e) 7, $r = 3.97478$, $\varphi = 224^\circ$; (f) 8, $r = 9.09021$, $\varphi = 170^\circ$. In each case at the end $q = \phi(\mathcal{E}_2) \approx 1$. Such deterministic geodesics of space-time–time can model (qualitatively, at least) quantum behavior of electrons in atoms, of 'quantum jumps' between electron orbits in particular.

to the sphere $S(\mathcal{E})$ centered at the spatial location of $\mathcal{E}$ and with the nonzero radius $\hat{R}e^{-t(\mathcal{E})}$. By definition $\mathcal{E} = \mathcal{S}(\mathcal{E})$, and therefore every test particle track that has $\mathcal{E}$ as one of its endpoints would include $S(\mathcal{E})$ by continuity. What is more, the full geodesic path is itself a one-parameter family of space-time–time points, thus of (hyper-hyperboloidal) 'spheres' of space-time. The 3-'sphere' S_ζ that is the point at $[\![\, x^\mu, \zeta \,]\!]$ has as its center the spatial 2-sphere that is the event at $[\![\, x^\mu \,]\!]$ in space-time. The radius of S_ζ is $\hat{R}e^{-\zeta}$, which goes to zero at $\mathcal{E}$. According to the space-time metric constructed in Fig. 1, S_ζ is the set of all 2-spheres that lie an angular distance $\hat{R}e^{-\zeta}$ from the central 2-sphere. This set is a union of disjoint subsets each of which is a one-parameter family of 2-spheres all mutually tangent to one another at a single point of the central 2-sphere, which they all intersect in an angle of radian measure $\hat{R}e^{-\zeta}$. These families, which in the conventional sense are null generators of the hyper-hyperboloid that is the 3-sphere S_ζ, are null geodesics of space-time which broadcast the location and size of the central 2-sphere, both forward in time and backward. As $\zeta \to \infty$ the 2-spheres all become tangent to the central sphere. Conventionally put, the hyperboloid S_ζ collapses to a null cone, whose vertex is the event corresponding to (i. e., equal to by definition) the limiting central sphere $S(\mathcal{E})$. The events on or interior to the past null cone of $\mathcal{E}_1$ and the future null cone of $\mathcal{E}_2$ receive no information about the particle, but every other event is notified of the particle's existence (by being an event on a null generator of one of the hyperboloids S_ζ in the space-time–time geodesic, thus by being a 2-sphere in one of the families of mutually tangent 2-spheres whose union is S_ζ). If we now consider the test particle following the path p to in fact *be* a 2-sphere in space, going forward in (primary) time by shrinking, then, because it has nonzero spatial radius at $\mathcal{E}_1$ and $\mathcal{E}_2$, the particle can be deemed to exist there, even though it is visible only between $\mathcal{E}_1$ and $\mathcal{E}_2$.

A second test particle whose track shared with that of the first an endpoint $\mathcal{E}$ would have in common with the first the 2-sphere $S(\mathcal{E})$, thus would be, for an instant at least, the same particle. The two particles could be thought of as extensions of one another, as well as of all other particles that shared the endpoint $\mathcal{E}$. Such an event would be an 'interaction' event, not unlike a vertex in a quantum mechanical Feynman diagram. Built in to the interaction would be that all participating charged particles have the same charge magnitude $|q| = \phi(\mathcal{E})$.

There is much left unreported here, but I trust that what has been reported is sufficient to lend credence to the proposition that some measure of the physics of quantum phenomena can be extracted, qualitatively and to some degree quantitatively, from the geometry of three-dimensional space.

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